Delay and Throughput of Network Coding with Path Redundancy for Wireless Mesh Networks

Nikolaos Pappas, Vasilios A. Siris, Apostolos Traganitis
Institute of Computer Science, Foundation for Research and Technology - Hellas (FORTH)
P.O. Box 1385, GR 711 10 Heraklion, Crete, Greece
{npapas,vsiris,tragani}@ics.forth.gr

Abstract—**In this paper we investigate the performance that can be achieved by exploiting path diversity through multipath network coding with redundancy. We extend our work on the analysis of the delay and throughput for multipath forwarding for the case of hop by hop retransmission instead of end to end in the previous work. A key contribution of the paper is the demonstration of the tradeoff between packet delay and throughput achieved by combining multipath forwarding and network coding, and comparison of this tradeoff with other routing schemes. The analytical framework considers the case of hop by hop retransmission for achieving reliability, and is generalized for an arbitrary number of paths and hops. The proposed framework consists of two parts, the first one considers end to end coding process and the second one hop by hop.**

I. Introduction

In this paper, we extend our work in [1]. In that paper we investigated the performance that can be achieved by exploiting path diversity through multipath forwarding for end to end retransmissions. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. Another result was that as the number of available paths increases, the gain from network coding also increases.

We consider unicast flows in a multi-hop wireless (mesh) network with lossy directional links. In such networks the largest percentage of uplink traffic is destined for or originates from a gateway interconnecting the mesh network to a wired network. Moreover, a mesh node can provide access to multiple clients. Hence, the uplink traffic from these clients that is destined to the same gateway can be coded at the mesh node, and decoded at the gateway. Similarly, downlink traffic destined for the clients of the same mesh node can be coded at the gateway and decoded at the mesh node. In real-world wireless scenarios, end-to-end connectivity is often intermittent, limiting the performance of end-to-end transport protocols. For this reason hop by hop retransmission is preferred [2] and this work will focus on hop by hop retransmission in the presence of link losses with either end to end or hop by hop network coding process.

The goal of this paper is to investigate the performance that can be achieved by exploiting path diversity through multipath forwarding and redundancy through network coding in a multi-hop network using hop by hop retransmission. Specifically, we compare the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, the transmission of multiple copies of a single flow over multiple paths (which achieves the least delay due to the highest redundancy), and traditional single path routing.

The idea of using redundancy is central in channel coding theory. In this work we use redundant paths to send coded packets in order to recover the loss of information using packets from another path, thus decreasing the delay. The work in [3] uses path diversity for fast recovery from link outages. The work in [4] introduces error correcting network coding as a generalization of classical error correcting codes. The work of [5] considers diversity coding, and investigates the allocation of data to multiple paths that maximizes the probability of successful reception. The work of [6] extends the previous work, in the case where the failure probabilities are different for different paths, and when the paths are not necessarily independent.

Our contribution and a key difference with the previous works is that we study the delay and throughput tradeoff and compare network coding with other transmission schemes such as single path, multipath and multicopy. We study the average delay per packet and the throughput achieved, disregarding the queueing delay at the sender, the encoding and decoding delays, and the ACK transmission delays. The model we assume is a one-source unicast acyclic network with lossy directional links. The analytical framework presented in this paper considers the case of hop by hop retransmission for achieving reliability, and is generalized for an arbitrary number of paths and hops. The coding process we study includes end to end and hop by hop coding.

The rest of the paper is organized as follows: Section II presents the network models assumed in the present paper. Sections III and IV presents the analytical model for the throughput and delay in the case of hop by hop retransmissions where the coding is end to end and hop by hop respectively.

This work was supported in part by the European Commission in the 7th Framework Programme through project EU-MESH (Enhanced, Ubiquitous, and Dependable Broadband Access using MESH Networks), ICT-215320, http://www.eu-mesh.eu

N. Pappas and A. Traganitis are also with the University of Crete. V. A. Siris is also with the Athens University of Economics and Business.

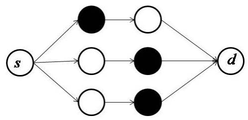


Fig. 1. An instance of a network with node-disjoint paths, with $n = 3$ and $m = 3$, the corresponding state is $S = (1, 2, 2)$.

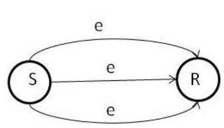
Section V presents the case of a network with three paths and different error probabilities. Section VI presents numerical results based on the previous models, and finally section VII concludes the paper.

II. The model of the network

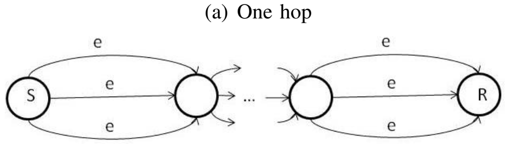
The model we assume is a one-source unicast acyclic network with lossy directional links. We consider the case of hop by hop retransmission. When an error occurs at the transmission between two nodes for example node i to $i+1$, node i re-sends the information to $i+1$. Figure 1 shows a network with node-disjoint paths where the coding process is end to end. Figure 2(b) presents a network with paths having nodes in common where the coding process is hop by hop. When the network has more than one hop, the inner nodes can decode the information and then re-encode it. In this work we study the average delay per packet and the throughput achieved, disregarding the queueing delay at the sender, the encoding and decoding delays, and the ACK transmission delays. For the sake of simplicity we assume that the number of hops is the same for every path in the network and every link has the same error probability e. In section V we relax this assumption and present the analysis of a network with three paths each with a different error probability.

III. Analytical model for node-disjoint paths (End to end coding)

Consider a source s and its receiver d. The network we study here has n paths, each path having m hops. The original packets are k (where $k \leq n$). In order to find the average time that is needed for d to receive the packets, we model our problem using absorbing Markov Chains [7]. The chain is absorbed when the receiver d has received k packets. A state of this chain is denoted by S. S is a n- tuple: $S = (s_1, s_2, ..., s_n)$, where s_i is the number of hops traversed by a packet on path i, note that $0 \leq s_i \leq m$ and $1 \leq i \leq n$. For example in Figure 1, the nodes with black color are the ones that have received already the packet.



(a) One hop



(b) n hops

Fig. 2. Simple network with three paths having nodes in common

The state space denoted by V_S contains all the $(m+1)^n$ states of the Markov Chain. V_S is divided into two sub-spaces V_T and V_A, $V_S = V_T \cup V_A$. V_T and V_A are the spaces that contain the transient and absorbing states respectively. There are $|V_S| = (m+1)^n$ states in total. The absorbing ones are:

$$|V_A| = \sum_{i=k}^{n} \binom{n}{i} \tag{1}$$

The transient states are:

$$|V_T| = (m+1)^n - \sum_{i=k}^{n} \binom{n}{i} \tag{2}$$

The transition matrix T of the Markov Chain has the following canonical form [7]:

$$T = \begin{pmatrix} P & R \\ 0 & I \end{pmatrix} \tag{3}$$

P is an $|V_T| \times |V_T|$ matrix, R is $|V_T| \times |V_A|$ and I is $|V_A| \times |V_A|$ matrix. It is known that for an absorbing Markov Chain the matrix $I - P$ has an inverse [7]. Also it is known that:

$$t = (I - P)^{-1} \mathbf{1}_{|V_T| \times 1} \tag{4}$$

where t is the expected number of steps before the chain is absorbed and $\mathbf{1}_{|V_T| \times 1}$ is the all-ones column vector. The first element of t is the expected time for the chain to be absorbed starting from the initial state, that is the delay we want to compute. The rest of this section presents the procedure in order to compute the matrix P. We assign indices for the transient states, the initial state $S_0 = (0, 0, ..., 0)$ being the first one. This indexing facilitates the computation of the elements of matrix P, for example P_{ij} is the probability of transition from $S_i = (s_1^i, ..., s_n^i)$ to $S_j = (s_1^j, ..., s_n^j)$. The elements of P can be computed by the following:

$$P_{ij} = \begin{cases} 0, & \text{if } \exists k \text{ s.t. } s_k^j < s_k^i \text{ or } s_k^j - s_k^i > 1 \\ e^{n-correct-final}(1-e)^{correct}, & \text{otherwise.} \end{cases}$$

$$final = \sum_{k=1}^{n} \lfloor \frac{s_k^i}{m} \rfloor \tag{5}$$

$$correct = \sum_{k=1}^{n} (s_k^j - s_k^i) \tag{6}$$

The chain is absorbed when the receiver has received at least k packets, which means $final \geq k$.

Next we show how the previous procedure can be applied for the computation of the delay and throughput for single path, multipath, multicopy and multipath with network coding.

A. Single Path

For this case, we apply the previous procedure with $n := 1$ and $k := 1$, to calculate the delay D_{sp}. The throughput is given by $Thr_{sp} = \frac{1}{D_{sp}}$.

B. Multipath

The delay for multipath is equal to D_{sp}. The throughput is given by $Thr_{mp} = \frac{n}{D_{sp}}$.

C. Multicopy

Multicopy is the technique for maximum redundancy, we send the same symbol to all paths. We apply the previous procedure with $n := n$ and $k := 1$, to calculate the delay for multicopy D_{mcop}. The throughput is given by $Thr_{mcop} = \frac{1}{D_{mcop}}$.

D. Multipath with Network Coding

There are n paths and we send k original(uncoded) symbols through n linear combinations (redundancy), the procedure is applied with parameter $n := n$ and $k := k$, to calculate the delay for network coding D_{nc}. The throughput is given by $Thr_{nc} = \frac{k}{D_{nc}}$.

In section VI we will present the arithmetic results derived from the previous procedure for various numbers of paths and hops.

IV. Analytical model for paths with nodes in common (Hop by hop coding)

The derivation of the equations in this section is based on [1] section II. There is only a small change when network coding is used.

A. Three paths

In this part we will present the equations corresponding to network depicted in Figure 2(a). The probability of error in each path is e.

1) Single Path: The average delay is given by $D_{sp} = \frac{1}{1-e}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}} = 1 - e$.

2) Multipath: Multipath has the same delay as the single path $D_{mp} = D_{sp}$ and its throughput is three times the throughput of single path $Thr_{mp} = 3Thr_{sp}$.

3) Multicopy: The delay and throughput are $D_{mcop} = \frac{1}{1-e^3}$ and $Thr_{mcop} = \frac{1}{D_{mcop}}$ respectively.

4) Multipath with Network Coding: The delay D_{nc} is the average delay to receive at least two of the three independent linear combinations sent by node S: $D_{nc} = \frac{(1-e)^3+3e(1-e)^2+3e^2(1-e)(1+D_1)+e^3}{1-e^3}$ where $D_1 = D_{mcop} = \frac{1}{1-e^3}$. The additional delay D_1 is to receive one more linear combination when we have already received one. Since in the time interval D_{nc} node R receives two data packets, the average throughput is given by $Thr_{nc} = \frac{2}{D_{nc}}$.

B. Seven paths

1) Single Path: The average delay is given by $D_{sp} = \frac{1}{1-e}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}} = 1 - e$.

2) Multipath: Multipath has the same delay as the single path $D_{mp} = D_{sp}$ and its throughput is seven times the throughput of the single path $Thr_{mp} = 7Thr_{sp}$.

3) Multicopy: The delay and throughput are $D_{mcop} = \frac{1}{1-e^7}$ and $Thr_{mcop} = \frac{1}{D_{mcop}}$ respectively.

4) Multipath with Network Coding: We have 3 packets to transmit through $2^3 - 1 = 7$ paths. According to lemma in appendix A in [1] we need at least 3 and at most 4 linear packet combinations to be able to decode the initial packets. The delay for receiving 3 or 4 linear combinations is denoted by D_{nc-L},D_{nc-U} respectively.

$$D_{nc-L} = \frac{1}{1-e^7}[\sum_{i=3}^{7}\binom{7}{i}(1-e)^i e^{7-i} + \sum_{i=1}^{2}\binom{7}{i}(1-e)^i e^{7-i}(1 + D_{3,3-i}) + e^7]\,,$$

where $D_{3,i}$ is the delay to receive $i = 1, 2$ encoded packets when 3 needed, $D_{3,1} = \frac{1}{1-e^7}$, $D_{3,2} = \frac{1}{1+e^7}[1 - e^7 + (1 + \frac{1}{1-e^7})(e^3(1-e^4) + e^4(1-e^3))]$ The average delay to receive 4 linear combinations is given by:

$$D_{nc-U} = \frac{1}{1-e^7}[\sum_{i=4}^{7}\binom{7}{i}(1-e)^i e^{7-i} + \sum_{i=1}^{3}\binom{7}{i}(1-e)^i e^{7-i}(1 + D_{4,4-i}) + e^7]\,,$$

where $D_{4,i}$ is the delay to receive $i = 1, 2, 3$ encoded packets when 4 needed, $D_{4,1} = D_{3,1}$, $D_{4,2} = D_{3,2}$, $D_{4,3} = D_{nc-L}$. The throughput is given by: $Thr_{nc} = \frac{3}{D_{nc}}$.

Note: If the network topology has n hops as in figure 2(b), then in order to find the total delay with the previous models we just need to add the delays for all the hops. In the case where all links have the same error probabilities then the total delay is n times the delay for one hop.

V. Analytical model for the network with three paths and one hop each with different link errors

In this section we will give the equations for the delay and throughput for the above routing schemes when then paths have different error probabilities. The derivation of the equations in this section is again based on [1] Appendix B. There is only a small change when network coding is used.

A. Single Path

The single path routing scheme selects the best available path from the three available. Thus the delay is $D_{sp} = \frac{1}{1-\min_i e_i}$ and the throughput is $Thr_{sp} = \frac{1}{D_{sp}}$.

B. Multipath

In this routing scheme different data flows follow different paths, so the average delay per packet and the throughput are: $D_{mp} = \frac{1}{3}\sum_{i=1}^{3}\frac{1}{1-e_i}$, $Thr_{mp} = \frac{3}{D_{mp}}$ respectively.

C. Multicopy

The multicopy scheme uses all available paths to forward the same flow, in this way achieves the maximum redundancy (but wasting resources). The average delay is: $D_{mcop} = 1/(1- \prod_{i=1}^{3} e_i)$ and the average throughput is: $Thr_{mc} = \frac{1}{D_{mc}}$.

D. Multipath with Network Coding

Multipath with Network Coding uses all available paths sending linear combinations of initial packets to each of them. In this case with three paths available, we encode two packets and there are three linear combinations. In order to decode the initial packets we have to receive two linear independent combinations. The average delay is given by:

$$D_{nc} = \frac{1}{1-\prod_{i=1}^{3} e_i}[\prod_{i=1}^{3}(1-e_i) + \sum_{i=1}^{3} e_i \prod_{j=1,j\neq i}^{3}(1-e_j) + \sum_{i=1}^{3}(1-e_i)(1+D_1)\prod_{j=1,j\neq i}^{3} e_j + \prod_{i=1}^{3} e_i]$$

where $D_1 = \frac{1}{1-\prod_{i=1}^{3} e_i}$.

Notice that for the Multipath with Network Coding scheme we are not able to compute the average delay per packet (because of the linear combinations) and we calculate the delay needed to receive at least two linear independent combinations. This means that the above delay is the delay to receive all the initial packets. The throughput is: $Thr_{mc} = \frac{2}{D_{nc}}$.

VI. Numerical Experiments

In this section we present arithmetic results based on the models described in the sections III, IV and V.

A. Results for networks with node disjoint paths (End to end coding)

Table I shows the delay - throughput tradeoff for networks with node disjoint pats. Multipath with network coding achieves delay which is smaller than single and multipath, but worst than multi-copy forwarding. The throughput achieved by multipath with network coding is better than this achieved by multicopy forwarding. The gain from network coding is not so much, about $7-9\%$ in terms of delay for the errors $e = 0.2$, $e = 0.4$ and three paths with two hops each.

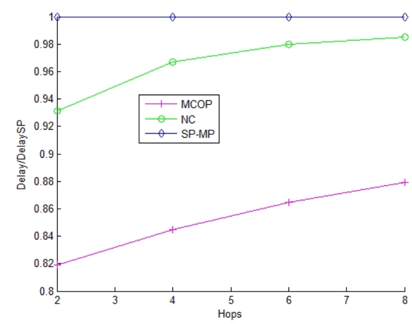
Multipath with network coding achieves delay, which is slightly better than single and multipath (about 4%), but worst than multi-copy forwarding for error probabilities 0.2 and 0.4 for the network with three paths and four hops. In comparing with two hops we observe that the gain for network coding is decreased. We can see that network coding approaches multipath in term of delay. This is expected because of the relatively small number of paths and packets.

Figures 3(a) and 3(b) show how the number of hops affects the delay and throughput compared to delay for single path and throughput for multipath respectively.

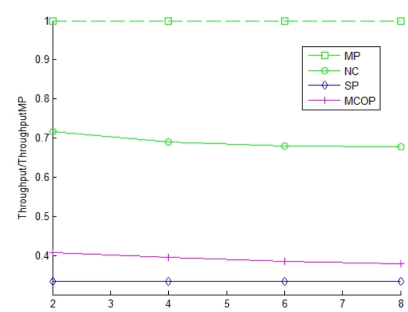
In the following we will show plots for the network with seven paths and two hops. Table I includes two lines for network coding, one corresponding to the case of decoding after receiving three linear combinations (which is denoted by NC-L) and one for decoding after receiving four (which is denoted by NC-U); These number represent the lower and upper bound of the number of coded packets required to retrieve all packets at the receiver, as indicated by lemma [1]. Multipath with network coding achieves delay, which is better than single and multipath (about 20%), but worst than multi-copy forwarding. In term of throughput network coding is much better(150%) than multicopy. Multicopy is superior when the loss become large and for a large number of hops because of its higher redundancy.

Scheme	Error	Paths	Hops	Delay/DelaySP	Thr/ThrSP
NC	0.2	3	2	0.9312	2.148
SP	0.2	3	2	1	1
MP	0.2	3	2	1	3
MCOP	0.2	3	2	0.819	1.221
NC	0.2	3	4	0.967	2.07
SP	0.2	3	4	1	1
MP	0.2	3	4	1	3
MCOP	0.2	3	4	0.845	1.184
NC	0.4	3	2	0.93	2.15
SP	0.4	3	2	1	1
MP	0.4	3	2	1	3
MCOP	0.4	3	2	0.694	1.44
NC	0.4	3	4	0.967	2.07
SP	0.4	3	4	1	1
MP	0.4	3	4	1	3
MCOP	0.4	3	4	0.761	1.31
NC-L	0.2	7	2	0.825	3.64
NC-U	0.2	7	2	0.888	3.38
SP	0.2	7	2	1	1
MP	0.2	7	2	1	7
MCOP	0.2	7	2	0.8	1.25
NC-L	0.4	7	2	0.771	3.89
NC-U	0.4	7	2	0.903	3.32
SP	0.4	7	2	1	1
MP	0.4	7	2	1	7
MCOP	0.4	7	2	0.613	1.63

TABLE I
DELAY-THROUGHPUT TRADEOFF FOR NODE DISJOINT PATHS



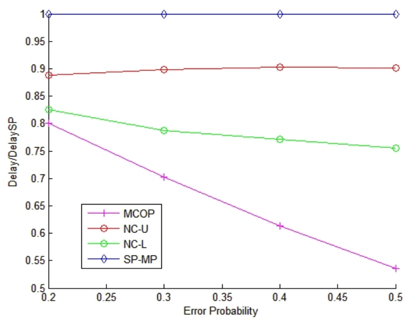
(a) D/D_{sp} vs number of hops



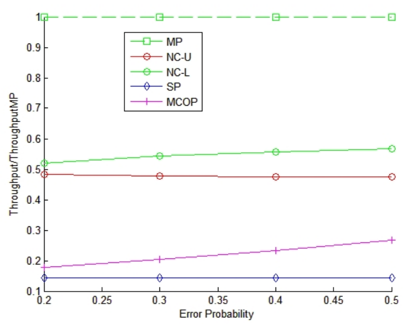
(b) Thr/Thr_{mp} vs number of hops

Fig. 3. Delay and throughput for a different number of hops, in the case of three paths and $e = 0.2$ (node disjoint paths)

Throughput achieved by multipath with network coding is better than that achieved by multi-copy routing. Figure 4(a) shows that, as expected, the improvement in terms of lower delay which is achieved by multipath with network coding and multi-copy increases not so much with increasing error probability. Regarding throughput, observe that a higher loss probability does affect the gains of multipath with network coding over single-path forwarding, as much they do in the case of multi-copy transmission; this is also shown in figure 4(b).



(a) D/D_{sp} vs e



(b) Thr/Thr_{mp} vs e

Fig. 4. Delay and throughput vs e, in the case of seven paths and two hops each (node disjoint paths)

B. *Results for networks with paths having common nodes (hop by hop coding)*

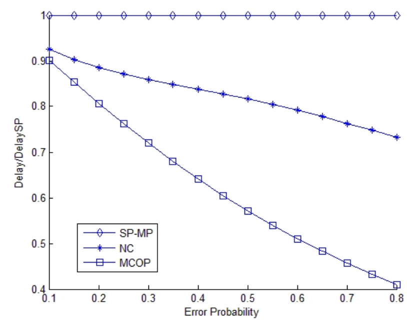
Figures 5(a) and 5(b) show how the error probability e affects the delay and throughput compared to delay for single path and throughput for multipath respectively for the network with three paths and hop by hop coding process. Figures 6(a) and 6(b) show show the how the error probability e affects the delay and throughput compared to delay for single path and throughput for multipath respectively for the network with seven paths. In these plots we see the advantage of network coding as error increases. In heavy noise the network coding outperforms even multipath in terms of throughput and it has only a fraction of delay of the singlepath (and multipath) scheme.

Table II shows the delay - throughput tradeoff the networks with paths having nodes in common for error probabilities $e = 0.2$ and $e = 0.4$. For the case of three paths multipath with network coding achieves delay, which is better than single and multipath (about $13 - 18\%$), but worst than multi-copy forwarding. In term of throughput network coding is much better(90%) than multicopy. For the case of seven paths multipath with network coding achieves delay, which is better than single and multipath (about $22-40\%$), but slightly worse than multi-copy forwarding. In term of throughput network coding is much better($200 - 350\%$) than multicopy.

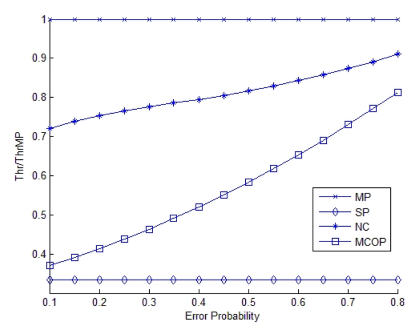
The above results indicate that the network coding in a network with paths with nodes in common has profound advantages compared to topologies with node-disjoint paths.

Scheme	Error	Paths	Delay/DelaySP	Thr/ThrSP
NC	0.2	3	0.8845	2.261
SP	0.2	3	1	1
MP	0.2	3	1	3
MCOP	0.2	3	0.807	1.24
NC	0.4	3	0.838	2.386
SP	0.4	3	1	1
MP	0.4	3	1	3
MCOP	0.4	3	0.641	1.56
NC-L	0.2	7	0.804	3.733
NC-U	0.2	7	0.827	3.629
SP	0.2	7	1	1
MP	0.2	7	1	7
MCOP	0.2	7	0.8	1.25
NC-L	0.4	7	0.656	4.573
NC-U	0.4	7	0.777	3.862
SP	0.4	7	1	1
MP	0.4	7	1	7
MCOP	0.4	7	0.601	1.664

TABLE II
DELAY-THROUGHPUT TRADEOFF FOR PATHS WITH NODE IN COMMON

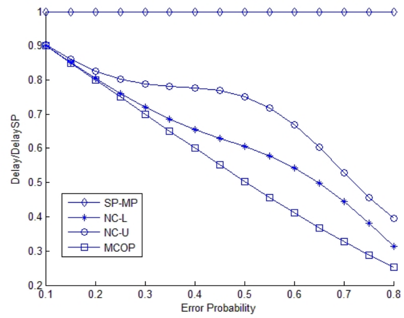


(a) D/D_{sp} vs e

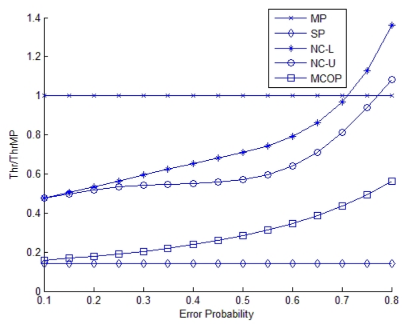


(b) Thr/Thr_{mp} vs e

Fig. 5. Delay and throughput vs e, in the case of three paths (paths with common nodes)



(a) D/D_{sp} vs e



(b) Thr/Thr_{mp} vs e

Fig. 6. Delay and throughput vs e, in the case of seven paths (paths with common nodes)

C. Results for Network with three paths with different error probabilities

Table III shows the delay-throughput trade-off for two different scenarios.

Scheme	e_1	e_2	e_2	Delay/DelaySP	Thr/ThrSP
NC	0.3	0.4	0.5	0.974	2.053
SP	0.3	0.4	0.5	1	1
MP	0.3	0.4	0.5	1.189	2.523
MCOP	0.3	0.4	0.5	0.745	1.343
NC	0.5	0.6	0.8	1.056	1.894
SP	0.5	0.6	0.8	1	1
MP	0.5	0.6	0.8	1.583	1.895
MCOP	0.5	0.6	0.8	0.658	1.52

TABLE III
DELAY-THROUGHPUT TRADEOFF FOR THREE PATHS WITH DIFFERENT ERROR PROBABILITIES

In the case of $e_1 = 0.5$, $e_2 = 0.6$ and $e_2 = 0.8$ the multipath with network coding is the superior routing scheme, has almost the same delay as the singlepath but the double throughput. Multipath has the same throughput with network coding but 60% more delay than single path.

Summarizing the above we can state that network coding offers significant advantages as the number of paths increases, when the nodes inside the network are able to decode and encode the received packets and finally under heavy noise environments.

VII. CONCLUSION

In this paper we investigated the performance and reliability that can be achieved by exploiting path diversity through multipath forwarding together with redundancy through network coding, when hop by hop retransmissions are used for achieving reliable packet transmission with end to end and hop by hop coding. We compared the performance and tradeoff in terms of packet delay and throughput achieved by combining multipath forwarding and network coding, with that of simple multipath routing of different flows, transmission of multiple copies of a single flow over multiple paths, and single path routing. We saw that network coding decreases the delay that is needed for the transmission of a packet compared with multipath and traditional single path forwarding, achieving a delay-throughput balance that lies between the corresponding performance of simple multipath and multicopy forwarding, which sends the same packet across all available paths. We saw that as the number of hops increases the gain for delay decreases for the network with node disjoint paths (end to end coding). Another important result is that as the number of available paths increases, the gain from network coding also increases. The significant advantages of network coding with redundancy appeared when hop by hop coding (paths with nodes in common) applied. Under heavy noise though the network coding scheme outerforms all the other routing schemes. This is obvious from the arithmetic results in the network with paths having different error probabilities.

The hop by hop coding process is not computationally expensive due to the linearity of the network coding technique and for this reason the delay from decoding and encoding is not so important.

The conclusion is that network coding offers significant advantages as the number of paths increases, when the nodes inside the network are able to decode and encode the received packets and finally under heavy noise environments.

Future work will investigate the delay - throughput tradeoff in the presence of bursty errors for hop by hop retransmissions. Another extension of this work should be the study of networks with different error probability for each hop for more complex topologies. Our future work involves the impact of interference and congestion to schemes described above.

REFERENCES

[1] N. Pappas, V. A. Siris, and A. Traganitis, "Path diversity gain with network coding and multipath transmission in wireless mesh networks," in *The Second IEEE WoWMoM Workshop on Hot Topics in Mesh Networking (accepted)*, 2010.

[2] S. Heimlicher, M. Karaliopoulos, H. Levy, and M. May, "End-to-end vs. hop-by-hop transport under intermittent connectivity," in *Autonomics '07: Proceedings of the 1st international conference on Autonomic computing and communication systems*, 2007.

[3] Ayanoglu, C.-L. I, R. Gitlin, and J. Mazo, "Diversity coding for transparent self-healing and fault-tolerantcommunication networks," in *IEEE Transactions on Communications*, pp. 1677 – 1686, 1993.

[4] R. W. Yeung and N. Cai, "Network error correction, part i: Basic concepts and upper bounds," in *Communications in Information and Systems*, pp. 19 – 36, 2006.

[5] A. Tsirigos and Z. Haas, "Analysis of multipath routing, part 1: The effect of packet delivery ratio," in *IEEE Transactions on Wireless Communications vol.3 no. 1*, pp. 138–146, 2004.

[6] A. Tsirigos and Z. Haas, "Analysis of multipath routing, part 2: Mitigation of the effects of frequently changing topologies," in *IEEE Transactions on Wireless Communications vol.3 no. 2*, pp. 500–511, 2004.

[7] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill Higher Education, 2002.