Stability and Performance Issues of a Relay Assisted Multiple Access Scheme

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***Abstract*—In this paper, we examine the operation of a node relaying packets from a number of users to a destination node. We assume that the relay does not have packets of its own, the traffic from the users is saturated and we have random access of the medium with slotted time. We study the impact of the relay node on the throughput per user and the aggregate throughput for the group of users. We obtain analytical expressions for the arrival and service rate of the queue of the relay, the stability conditions and the average length of the queue. We quantify the above, analytically and through simulations, for different numbers of users and different transmission characteristics of the users and the relay and give the conditions under which there are significant advantages from the deployment of the relay.**

I. INTRODUCTION

Recently, the relay channel has attracted attention from the wireless communications community. The relay nodes inside the network can help to exploit spatial diversity in cooperative communications and thus lead to better data transmission from a source to a destination node [1]. The classical relay channel was introduced by van der Meulen [2]. In [3], Cover and El Gamal determined the Shannon capacity region for the degraded relay channel. Recent work in [4] investigated the impact of cooperative communications at the multiple access layers with an introduction of cognitive multiple access protocol in the presence of a relay in the network. The authors in [5] proposed relay assisted protocols based on network coding and they have shown that for a single source single relay system with two destinations, the use of network coding at the relay increases the stable throughput. The works in [6] and [7] have studied information theoretic aspects of a special case of the relay channel called multi way relay. In [7] the authors consider clusters of multiple nodes which want to exchange information among them and clusters communicating simultaneously over a single relay terminal. In [8], the authors studied the network protocol-level cooperation at a three node network, one source, a relay with packets of its own and a destination over erasure channels; they proposed a cooperation strategy and they characterized the stability and the throughput region.

In this work we examine the operation of a node relaying packets from a number of users to a destination node. We assume that we have random access of the medium due to difficulty of global coordination, the time is slotted, and each packet transmission takes one time slot. The wireless channel between any two nodes in the network is modeled as a Rayleigh narrowband flat-fading channel with additive Gaussian noise, thus having a specific outage probability. We also assume that acknowledgements (ACKs) are instantaneous and error free. The relay does not have packets of its own and the traffic from the users is saturated. We obtain analytical expressions for the arrival and service rate of the relay's queue, the stability condition and the average length of the queue as functions of the probabilities of transmissions and the outage probabilities of the links. We study the impact of the relay node on the throughput per user and the aggregate throughput for the group of users when the queue of the relay is stable. We show that the throughput per user does not depend on the probability of the relay transmissions and that there is an optimum number of users that maximizes the aggregate throughput. This result is useful in determining the optimum size of a set of users that the relay could serve for maximizing the aggregate throughput or in a scenario with multiple relays (for example in a sensor network) where it can provide a way to distribute the users among the relays. All these analytical results have been verified with simulations.

The rest of the paper is organized as follows. Section II describes the system model. In Section III.A-C we study the characteristics of the relay's queue and we derive the analytical equations. In section III.D we derive the equations for the throughput per user and the aggregate throughout. We present the arithmetic and simulation results in Section IV and finally our conclusion are given in Section V.

II. SYSTEM MODEL

We consider a network with N users-sources, one relay node and a single destination node. The sources transmit packets to the destination with the *cooperation* of the relay; the case of $N=2$ is depicted in figure 1. The queues of the users are saturated; the relay does not have packets of its own, and just forwards the packets that have been received by the users. The relay node stores a packet in its queue when a transmission from the packet's source to the destination node has failed. We assume that we have random access of the medium (no

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coordination), that time is slotted, that each packet transmission occupies one time slot and the acknowledgements (ACKs) are instantaneous and error free. We introduce the following notation: A_i^t denotes the event that node *i* attempts to transmit at time slot t, with probability $P(A_i^t) = q_i$. Similarly A_0^t denotes the event that the relay node attempts transmission at time slot t when the queue is not empty, which is denoted by the event $\{Q^t > 0\}$. The probability that the relay node attempts transmission at time slot t is given by $P(A_0^t \cap \{Q^t > 0\}) = q_0 P(Q^t > 0)$. We assume that each node has a single transceiver; we also assume that simultaneous transmissions from more than one nodes will result in a collision. The "back-off" probabilities q_i reduce the number of these collisions, and, as we see in the following, determine the stability of the queue and the performance of the relaying system.

The wireless channel between any two nodes in the network is modeled as a Rayleigh narrowband flat-fading channel with additive Gaussian noise, thus having a specific outage probability, which can be derived as follows [10]: The received SNR_{ij} for the link between nodes i and j is $SNR_{ij} = |h_{ij}|^2 r_{ij}^{-\gamma} G / n_0$ where $|h_{ij}|^2$ is the square of the magnitude of the channel gain and has an exponential distribution with unity mean [10], G is the transmission power, r_{ij} is the distance between i and j, γ is the path loss exponent and n_0 is the variance of the additive white Gaussian noise. An outage event between nodes i and j operating with an SNR threshold equal to β is denoted by: $O_{ij} = \{h_{ij} : SNR_{ij} < \beta\} = \left\{h_{ij} : |h_{ij}|^2 < \beta \cdot n_0 \cdot r_{ij}^{\gamma} / G\right\}$. Therefore the probability of outage in the link between nodes i and j with an SNR threshold equal to β, is given by:

$$\Pr(O_{ij}) = \Pr\left(SNR_{ij} < \beta\right) = 1 - \exp\left(-\beta \cdot n_0 \cdot r_{ij}^{\gamma} / G\right)$$

since the distribution of the SNR is exponential. We denote by p_{ij} the success probability (no outage) of a link with SNR threshold equal to β; that is $p_{ij} = \Pr(\overline{O_{ij}}) = \exp(-\beta \cdot n_0 \cdot r_{ij}^{\gamma} / G)$.

The random variable X^t equals 1 when there is addition of a packet in the queue of the relay at time slot t, and is 0 otherwise. Similarly the random variable Y^t is 1 when there is removal of a packet from the relay queue at time slot t, and is 0 otherwise. That is:

$$X^t = 1\left[\bigcup_{i=1}^{N}\left\{A_i^t \cap \overline{A}_0^t\left(\bigcap_{j=1, j\neq 1}^{N} \overline{A}_j^t\right) \cap O_{id}^t \cap \overline{O}_{i0}^t\right\}\right]$$

$$Y^t = 1\left[A_0^t \cap \left\{\bigcap_{i=1}^{N} \overline{A}_i^t\right\} \cap \overline{O}_{0d}^t\right]$$

The queue size Q of the relay at time slot t+1 can be described using the above random variables by the following equation:

$$Q^{t+1} = (Q^t - Y^t)^+ + X^t, where\ (x)^+ = \max(0, x)$$

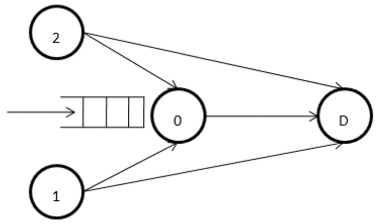

Fig. 1 Relay node with N=2 user nodes

III. ANALYSIS

In this section we will derive analytical equations for the characteristics of the relay's queue such as the arrival and service rates, the stability conditions, and the average queue length.

A. Computation of Arrival and Service Rate

If the queue of the relay is empty the arrival rate is denoted by λ_0 and by λ_1 if it is not; thus we have the following equation for the mean arrival rate λ:

$$\lambda = P(Q=0)\lambda_0 + P(Q>0)\lambda_1 = \\ = P(Q=0)\lambda_0 + (1 - P(Q=0))\lambda_1 \quad (1)$$

If the queue of the relay is empty then the relay, naturally, does not attempt to transmit, thus the probability of arrival λ_0 is :

$$\lambda_0 = \sum_{i=1}^{N} q_i (1 - p_{id}) p_{i0} \prod_{j=1, j\neq i}^{N} (1 - q_j) \quad (2)$$

If the queue is not empty then the arrival rate is given by:

$$\lambda_1 = (1 - q_0)\lambda_0 \quad (3)$$

The service rate is

$$\mu = P(Y^t = 1) = q_0 p_{0d} \prod_{i=1}^{N} (1 - q_i)$$

The probability that the queue in the relay at time slot t is not empty is given by:

$$P(Q^t > 0) = 1 - P(Q^t = 0) \quad (4)$$

In Figure 2 we present the discrete time Markov Chain with infinite states which describes the queue evolution.

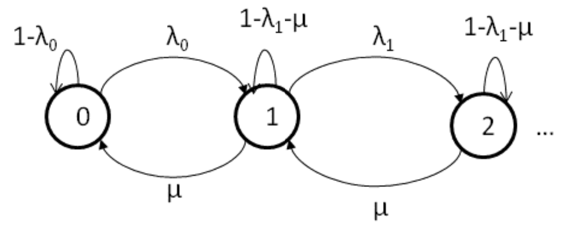

Fig. 2 The Markov Chain model of the queue

In order to compute the probability that the queue in the relay is empty, we will utilize the balance equations [11]. The stationary distribution π of the Markov Chain is computed by the balance equations, with $\pi(i)$ denoting the probability of state i when the chain is in the steady state. We have that:

$$\lambda_0\pi(0) = \mu\pi(1) \Leftrightarrow \pi(1) = \frac{\lambda_0}{\mu}\pi(0)$$

$$\pi(1)(\lambda_1 + \mu) = \lambda_0\pi(0) + \mu\pi(2) \Leftrightarrow \pi(2) = \frac{\lambda_1\lambda_0}{\mu^2}\pi(0)$$

$$\pi(n) = \frac{\lambda_0\lambda_1^{n-1}}{\mu^n}\pi(0)$$

It is well known [11] that

$$\sum_{n=1}^{\infty}\pi(n) = 1 \Leftrightarrow \pi(0) + \pi(0)\sum_{n=1}^{\infty}\frac{\lambda_0\lambda_1^{n-1}}{\mu^n} = 1$$

Therefore $\pi(0) = P(Q=0) = \dfrac{\mu-\lambda_1}{\mu-\lambda_1+\lambda_0}$ iff $\lambda_1 < \mu$ (5)

From Equations (2), (3), (5) and (1) above we can calculate the arrival rate λ.

$$\lambda = P(Q=0)\lambda_0 + (1-P(Q=0))\lambda_1 = \frac{\mu\lambda_0}{\mu-\lambda_1+\lambda_0}$$

$$\lambda = \frac{q_0 p_{0d}\prod_{i=1}^{N}(1-q_i)\left[\sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j)\right]}{q_0 p_{0d}\prod_{i=1}^{N}(1-q_i) + q_0\sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j)} \Rightarrow$$

$$\lambda = \frac{p_{0d}\prod_{i=1}^{N}(1-q_i)\left[\sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j)\right]}{p_{0d}\prod_{i=1}^{N}(1-q_i) + \sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j)}$$

From the expression of λ it is clear that the average arrival rate does not depend on q_0, the probability of transmission of the relay (despite the collisions).

B. Condition for the stability of the queue

An important tool to determine stability is Loyne's criterion [9], which states that if the arrival and service processes of a queue are strictly stationary and ergodic, the queue is stable if and only if the average arrival rate is strictly less than the average service rate. If the queue is stable, the departure rate (throughput) is equal to the arrival rate. The procedure for finding the q_0 for which the queue is stable is the following:

$$\frac{\lambda}{\mu} < 1 \Leftrightarrow \lambda < \mu \Leftrightarrow \frac{\mu\lambda_0}{\mu-\lambda_1+\lambda_0} < \mu \Leftrightarrow \frac{\lambda_0}{\mu-\lambda_1+\lambda_0} < 1 \Leftrightarrow$$
$$\Leftrightarrow \lambda_0 < \mu - \lambda_1 + \lambda_0 \Leftrightarrow \lambda_1 < \mu \Leftrightarrow$$
$$\Leftrightarrow (1-q_0)\lambda_0 < q_0 p_{0d}\prod_{i=1}^{N}(1-q_i)$$

$$\text{So } q_{0\min} = \frac{\sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j)}{\sum_{i=1}^{N}q_i(1-p_{id})p_{i0}\prod_{j=1,j\neq i}^{N}(1-q_j) + p_{0d}\prod_{i=1}^{N}(1-q_i)}$$

Thus the queue is stable if q_0 satisfies the inequality

$$q_{0\min} < q_0 < 1 \qquad (6)$$

Notice that the conditions $\lambda/\mu < 1$ and $\lambda_1/\mu < 1$ are equivalent in our model as we have shown above.

C. Average Queue Size

In order to compute the average queue size (when the queue is stable) we work as follows:

$$\overline{Q} = \sum_{i=0}^{\infty} i\pi(i) = \pi(0)\frac{\lambda_0}{\mu}\sum_{i=1}^{\infty} i(\lambda_1/\mu)^{i-1} =$$
$$= \pi(0)\frac{\lambda_0}{\mu}\left(\sum_{i=1}^{\infty}(\lambda_1/\mu)^i\right)' = \pi(0)\frac{\lambda_0}{\mu}\left(\frac{1}{1-\lambda_1/\mu} - 1\right)' =$$
$$= \pi(0)\frac{\lambda_0}{\mu}\left(\frac{1}{1-\lambda_1/\mu}\right)^2 = \frac{\mu\lambda_0}{(\mu-\lambda_1+\lambda_0)(\mu-\lambda_1)}$$

D. The throughput per user and the aggregate throughput

The throughput rate μ_i for the user i is given by the expression:

$$\mu_i = q_i(1-q_0P(Q>0))\prod_{j=1,j\neq i}^{N}(1-q_j)[p_{id} + (1-p_{id})p_{i0}] \quad (7)$$
$$\forall i = 1,...,N$$

In the previous equation we assume that the queue is stable so that the arrival rate from each user to the queue is the contributed throughput from it. Notice that after substituting P(Q>0) in this expression from Equations (4) and (5) we can see that the throughput is independent from q_0 as long as it is in the stability region (6). The aggregate throughput is given by $\mu_{total} = \sum_{i=1}^{N}\mu_i$ (8).

If we consider the previous network without the relay node then the throughput rates for user i is the following:

$$\mu_i = q_i\prod_{j=1,j\neq i}^{N}(1-q_j)p_{id} \quad \forall i = 1,...,N \qquad (9)$$

IV. ARITHMETIC RESULTS

In this section we present arithmetic results for the aggregate throughput and the range for the transmission probability of a stable relay queue based on Equations (1), (6) and (8) of the previous section. To simplify the presentation we consider the case where all the users have the same link characteristics and transmission probabilities, that is:

$$p_{i0} > p_{id}, \quad p_{id} = p_{jd}, \forall \text{i,j}, \quad q = q_i, \ i = 1,...,N$$

The results presented below have been verified with extensive simulations which confirmed the accuracy of the analysis in the previous sections.

A. Properties of the relay's queue for the case of N=2 users

Figure 3 presents the probability of an empty relay queue as the q_0 varies for various user link conditions and various user transmission probabilities q_i. As the relay transmission probability q_0 increases the queue is more likely to be empty (as expected). Equally expected is the decrease of the average queue size as q_0 increases, see figure 4. Two factors come into play as q_0 increases: a) the collisions are increasing and thus the arrivals are decreasing and b) the departures are increasing.

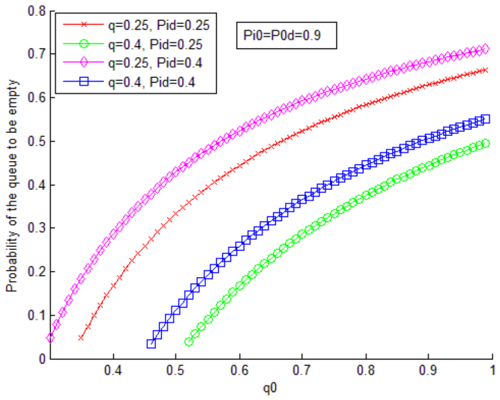

Fig. 3 Probability of empty queue vs relay transmission probability q_0

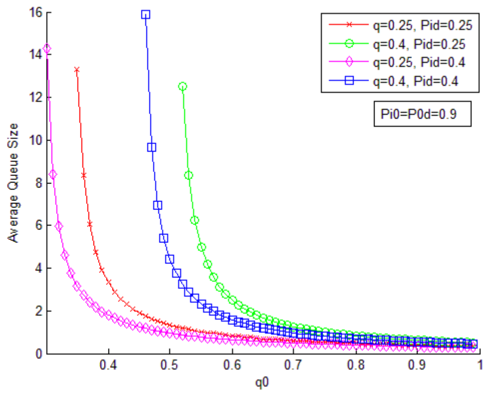

Fig. 4 Average queue size vs relay transmission probability q_0

B. The impact of the number of users

Figures 5a and 5b show the aggregate throughput versus the number of users. The figures show that the relay offers a significant advantage (up to almost 150% higher aggregate throughput) compared to the network without the relay. An interesting result is that, given the link characteristics and the transmission probabilities, there is an optimum number N^* of users that maximizes the aggregate throughput. This number could be used as a criterion for finding the optimum size of a subset of users served by the relay. This result will be also useful in a network with multiple relays for determining the way to allocate the users among the relays. Another interesting observation is that the relay ceases to offer an advantage if the number of users increases beyond a limit N_{max} which increases as the user-destination link quality deteriorates.

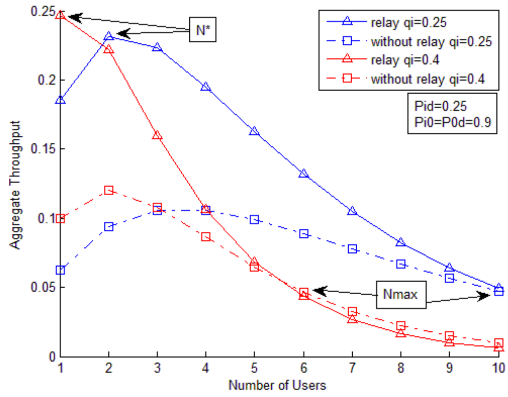

Fig. 5a Aggregate throughput vs the number of users: Poor user-destination link

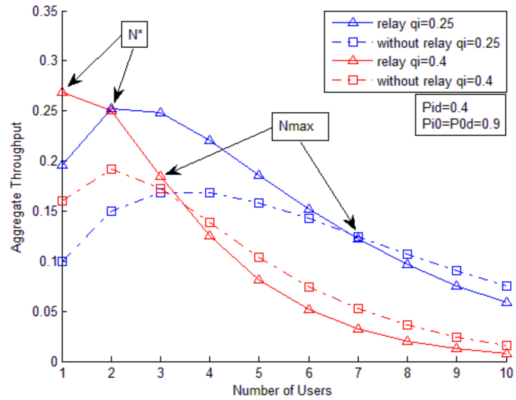

Fig. 5b Aggregate throughput vs the number of users: Better user-destination link

Figures 6a and 6b show the aggregate throughput versus the number of the users served by the relay for several user transmission probabilities. It is interesting to note that as the probability of transmission q_i =q increases the number of users for achieving the maximum aggregate throughput is decreasing**.**

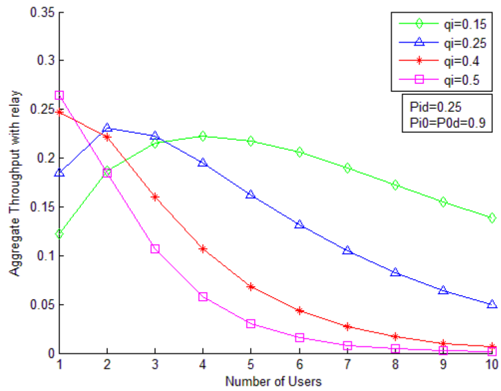

Fig. 6a Aggregate throughput with relay vs the number of users: Poor user-destination link

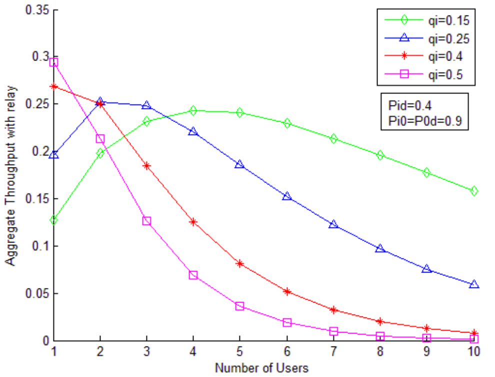

Fig. 6b Aggregate throughput with relay vs the number of users: Better user-destination link

In Figures 7a and 7b we present the minimum values q_{0min} of the relay transmission probability q_0 that lead to a stable queue versus the number of users. As expected q_{0min} is higher as the number increases, since there is need for more frequent relay transmissions.

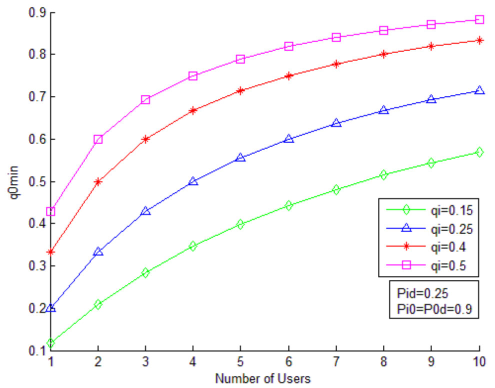

Fig. 7a q_{0min} vs number of users: Poor user-destination link

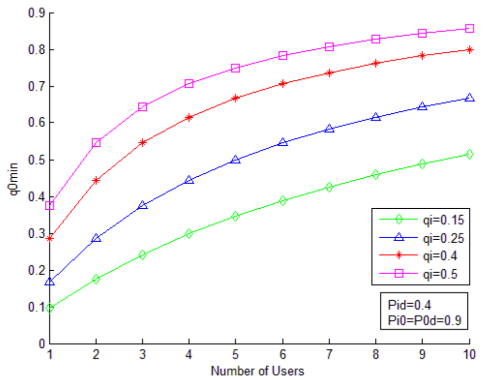

Fig. 7b q_{0min} vs number of users: Better user-destination link

V. CONCLUSION

In this paper, we examined the operation of a node relaying packets from a number of users to a destination node. We obtained analytical expressions for the relay's queue characteristics such as stability condition, arrival and service rate, average size. We showed that the arrival rate at the queue, despite the collisions with the transmissions of the relay, is independent of the relay probability of transmission, when the queue is stable. We studied the throughput per user and the aggregate throughput, and found that, under stability conditions, the throughput per user does not depend on the relay probability of transmission. All these analytical results have been verified with simulations. In Section IV we have given the conditions under which the utilization of the relay offers significant advantages. An interesting result is that, given the link characteristics and the transmission probabilities, there is an optimum number of users that maximizes the aggregate throughput. In addition there is a maximum number of users beyond which the utilization of the relay offers no advantage in terms of aggregate throughput. These results could be useful in a network with many users and multiple relays (for example in a sensor network) for determining the way to allocate the users among the relays. Future extensions of this work will include users with non-saturated queues, relay with its own packets and priorities for the users. An interesting aspect to study is the impact of scheduled access and a comparison with the random access of the medium. A step further could be a scenario with multiple relays and cooperation among them.

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